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# Analysis of the optical spectra and paramagnetic susceptibility of DyOF 

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#### Abstract

The optical absorption spectra of the rhombohedral DyOF were measured at selected temperatures between 9 and 300 K . The emission spectra of the $\mathrm{Dy}^{3+}$ ion in the LaOF and GdOF matrices were obtained at 77 K and room temperature. The energy level scheme of Dy ${ }^{3+}\left(4 \mathrm{f}^{9}\right.$ electron configuration) was simulated with a Hamiltonian of 20 parameters. The diagonalization of the energy matrices including simultaneously the free ion and crystal field (c.f.) interactions was carried out for the $\mathrm{C}_{3 \mathrm{v}}$ point symmetry of the $\mathrm{RE}^{3+}$ site. Good correlation with an rms deviation of $17 \mathrm{~cm}^{-1}$ was obtained between the experimental and calculated energy level schemes of 153 Kramers doublets. The c.f. parameters show only slight distortion from $\mathrm{O}_{\mathrm{h}}$ symmetry. A comparison to $\mathrm{Pr}^{3+}\left(4 \mathrm{f}^{2}\right), \mathrm{Nd}^{3+}\left(4 \mathrm{f}^{3}\right), \mathrm{Sm}^{3+}\left(4 \mathrm{f}^{5}\right), \mathrm{Eu}^{3+}\left(4 \mathrm{f}^{6}\right)$, and $\mathrm{Tb}^{3+}\left(4 \mathrm{f}^{8}\right.$ electron configuration) in other REOF matrices showed smooth evolution of the c.f. effect which was discussed in terms of the possible interactions involved. The x-ray powder diffraction pattern of DyOF between $6.5^{\circ}<2 \theta<120^{\circ}$ was analysed by the Rietveld profile refinement method. The structural data were used to calculate the c.f. parameters by the modified electrostatic point charge model yielding $B_{0}^{4}, B_{3}^{4}$ and $B_{0}^{6}$ values close to the experimental ones, whereas the $B_{0}^{2}$, $B_{3}^{6}$, and $B_{6}^{6}$ values were too large. Using the experimental free ion and c.f. wave functions, the paramagnetic susceptibility of DyOF as a function of temperature was simulated. Above the Néel temperature ( 3.6 K ), good agreement was obtained between the calculated and experimental susceptibilities in the paramagnetic range.


## 1. Introduction

The bulk of our knowledge of the electronic level structure of the rare earth (RE) ions $\left(4 f^{N}\right.$ electron configurations) has been obtained by optical spectroscopy. Even in the solid state, the 4 f electrons of the $\mathrm{RE}^{3+}$ ions are well protected from the ligand interaction and, consequently, they interact only weakly with the electrons of neighbouring atoms giving spectral and other physical properties characteristic of the particular $\mathrm{RE}^{3+}$ ion.

The $\mathrm{Dy}^{3+}$ ion has a $4 \mathrm{f}^{9}$ electron configuration which results in one of the most complicated energy level schemes among all $\mathrm{RE}^{3+}$ ions. The $4 \mathrm{f}^{9}$ configuration is characterized by 198 SLJ manifolds which split in the presence of the crystal field (c.f.) interaction into a total of 1001 sublevels (Stark levels). The low-lying ${ }^{6} \mathrm{H}$ and ${ }^{6} \mathrm{~F}$ multiplets

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form rather isolated groups up to $\sim 20000 \mathrm{~cm}^{-1}$ [1] above which the density of the states becomes high and the mixing of states with the same and different $J$ value increases. Because of the difficult interpretation of the optical spectra, only a few detailed spectroscopic studies concerning the $\mathrm{Dy}^{3+}$ ion in inorganic matrices have been carried out so far. The energy level scheme of $\mathrm{Dy}^{3+}$ has been analysed by using a phenomenological model including separately the free ion and c.f. effects for the simple fluoride $\mathrm{LaF}_{3}: \mathrm{Dy}^{3+}$ [13], and chloride $\mathrm{LaCl}_{3}: \mathrm{Dy}^{3+}[4,5]$ systems. Some c.f. calculations using a small basic set of c.f. levels have been performed for $\mathrm{Y}_{2} \mathrm{O}_{3}: \mathrm{Dy}^{3+}[6,7], \mathrm{YAlO}_{3}: \mathrm{Dy}^{3+}$ [8], $\mathrm{Dy}_{2} \mathrm{Ge}_{2} \mathrm{O}_{7}$ [9], elpasolites [10], $\mathrm{LiYF}_{4}: \mathrm{Dy}^{3+}$ [11], $\mathrm{CaWO}_{4}: \mathrm{Dy}^{3+}$ [12], $\mathrm{CaS}: \mathrm{Dy}^{3+}$ [13], and $\mathrm{Dy}_{3} \mathrm{Ga}_{5} \mathrm{O}_{12}$ [14].

The rare earth oxyhalides, $\mathrm{REOX}(\mathrm{X}=\mathrm{F}, \mathrm{Cl}, \mathrm{Br}$, and I$)$, offer a versatile system where an interchange of both the cation and anion leads to interesting structural modifications. This series provides an opportunity for a systematic study of the c.f. effect in both the RE and halide series, too. The stoichiometric RE oxyfluorides, REOF ( $\mathrm{RE}=\mathrm{La}-\mathrm{Lu}$ and Y ) [15-17] have trigonal symmetry due to the three-dimensionally linked $\mathrm{ORE}_{4}$ units [16]. The heavier $\operatorname{REOF}(\mathrm{RE}=\mathrm{Tm}, \mathrm{Yb}$, and Lu$)$ seem to have also a monoclinic structure [18]. The nonstoichiometric oxyfluorides, $\mathrm{REO}_{1-x} \mathrm{~F}_{1+2 x}$, for example $\mathrm{LaO}_{0.65} \mathrm{~F}_{1.70}$ [19], have (distorted) tetragonal symmetry due to the two-dimensional network of the $\mathrm{ORE}_{4}$ units.

In this work, the structure of the stoichiometric hexagonal DyOF was refined from x-ray powder diffraction data by using the Rietveld method. With those results, the c.f. parameters were calculated by using a modified electrostatic point charge model (PCEM). The energy level scheme of the $\mathrm{Dy}^{3+}$ ion in DyOF was also deduced from the optical absorption and luminescence spectra. The experimental energy level scheme was simulated with a phenomenological model using 14 free ion parameters describing the electrostatic and interconfigurational interactions as well as the spin-orbit coupling. The c.f. effect was simultaneously taken into account by six $B_{q}^{k}$ parameters. The phenomenological c.f. parameters were compared to those obtained from the PCEM calculations. From the c.f. parameter values the extent of the distortion from the ideal cubic fluorite-type structure was deduced. The evolution of the c.f. effect in the REOF: $\mathrm{RE}^{3+}$ series $\left(\mathrm{RE}^{3+}=\operatorname{Pr}^{3+}\right.$, $\mathrm{Nd}^{3+}, \mathrm{Sm}^{3+}, \mathrm{Eu}^{3+}, \mathrm{Tb}^{3+}$, and $\mathrm{Dy}^{3+}$ ) [20-24] was analysed in terms of the interactions possibly involved. The paramagnetic susceptibility of DyOF as a function of temperature was calculated using the wave functions obtained from the energy level simulation and was compared to the experimental values.

## 2. Experimental details

### 2.1. Sample preparation

The polycrystalline REOF samples were prepared by the solid state reaction between $\mathrm{RE}_{2} \mathrm{O}_{3}$ and $\mathrm{NH}_{4} \mathrm{~F}$ [25]. An excess of $\mathrm{NH}_{4} \mathrm{~F}\left(\mathrm{NH}_{4} \mathrm{~F} / \mathrm{Dy}_{2} \mathrm{O}_{3}\right.$ ratio $\left.=4.5\right)$ was utilized while heating the reactants at $860^{\circ} \mathrm{C}$ for 1.5 h in order to obtain the stoichiometric DyOF. For the luminescence measurements, the LaOF and GdOF hosts were doped with $1 \mathrm{~mol} \% \mathrm{of}^{3+}$. The reaction temperature was 1050 and $950^{\circ} \mathrm{C}$ for LaOF and GdOF, respectively, and a stoichiometric $\mathrm{NH}_{4} \mathrm{~F} / \mathrm{RE}_{2} \mathrm{O}_{3}$ ratio $(=2.00)$ was used. All samples were confirmed by routine x-ray powder diffraction to be of the correct stoichiometric REOF form. No presence of the nonstoichiometric phase of excess fluoride could be observed. The absence of additional lines in the optical spectra indicated that no additional phases were present.

### 2.2. X-ray powder diffraction measurements

The diffraction data of pure DyOF were collected at room temperature with an Enraf-Nonius PDS120 x-ray powder diffractometer using an INEL position sensitive detector. The XRD patterns for structure refinement were measured in the $2 \theta$ region between 6.5 and $120^{\circ}$. The monochromatic $\mathrm{Cu} \mathrm{K}_{\alpha_{1}}$ line ( $\lambda=1.540598 \AA$ ) was used as a radiation source.

### 2.3. Optical measurements

The luminescence spectra of the $\mathrm{Dy}^{3+}$ doped LaOF and GdOF were recorded at 77 and 300 K in the visible region between 500 and 800 nm . The luminescence was excited with the $\mathrm{Ar}^{+}$lines at 457.9 and 488.0 nm of a Carl Zeiss Jena ILA 120-1 laser, dispersed by a 1 m Czerny-Tuner type Jobin Yvon monochromator and detected by Hamamatsu R950 and R406 photomultipliers equipped with standard electronics. The resolution of the equipment was around $1 \mathrm{~cm}^{-1}$.

The optical absorption spectra of the pure hexagonal DyOF between 280 and 1700 nm were measured at selected temperatures between 9 and 300 K by a Cary 5E UV-vis-NIR spectrophotometer. The instrument was equipped with auto-calibration of the wavelength scale which resulted in reproducibility of the measurements better than $2 \AA$. The band width of $0.6 \AA$ used in measurements was sufficient taking into account the width of the absorption lines. The samples were prepared by adding 10 to 15 mass \% of DyOF to KBr and pressing a transparent disc of a thickness of 1 mm .

### 2.4. Magnetic measurements

The magnetic susceptibility measurements were carried out using a Quantum Design SQUID magnetometer. The available temperature range was from 1.7 to 400 K and the magnetic field used was 1000 Oe. The apparatus was calibrated using metallic Pd and $\mathrm{Hg}\left[\mathrm{Co}(\mathrm{SCN})_{4}\right]$ as standards. The molar magnetic susceptibility of DyOF was corrected for the ionic diamagnetism with the values $-12 \times 10^{-6},-11 \times 10^{-6}$, and $-19 \times 10^{-6} \mathrm{emu} \mathrm{mol}^{-1}$ for $\mathrm{O}^{2-}, \mathrm{F}^{-}$, and $\mathrm{Dy}^{3+}$, respectively [26].

## 3. Results and discussion

### 3.1. Crystal structure

The powder XRD data for DyOF were analysed with the Rietveld profile refinement method using the program DBSW-9006PC [27]. The unit cell, atomic positional parameters, thermal factors, and profile coefficients were all refined. The Gaussian peak shape modified for peak asymmetry was applied to the data in the $17^{\circ} \leqslant 2 \theta \leqslant 120^{\circ}$ range. The refined unit cell and atomic positional parameters for YOF [17] were used as starting values for the refinement. The refinement was carried out to a constant value ( $5.5 \%$ ) of the weighted pattern $R$-factor, $R_{\mathrm{wp}}$.

The structure of the stoichiometric DyOF belongs to the hexagonal (or rhombohedral) crystal system with $R \overline{3} m-\mathrm{D}_{3 \mathrm{~d}}^{5}$ (No 166 in [28]; $Z=6$ ) as the space group. The hexagonal unit cell dimensions found in our calculations are $a=3.7954(1)$ and $c=18.8892$ (2) $\AA$. All atoms lie in a special position with $x=y=0$ while the $z$ coordinates are as follows: $z_{\mathrm{RE}}=0.24144(2), z_{\mathrm{O}}=0.11946(10)$, and $z_{\mathrm{F}}=0.36877(2)$. The positional parameter values are close to those for the ideal cubic structure, $z_{\mathrm{RE}}=0.250, z_{\mathrm{O}}=0.125$, and $z_{\mathrm{F}}=0.375$ [17].

The coordination polyhedron around the RE atom is a bicapped trigonal antiprism with the RE-F distances $(1 \times 2.405(20)$ and $3 \times 2.437(2) \AA)$ longer than the RE-O distances $(1 \times 2.304(21)$ and $3 \times 2.252(5) \AA)$. The distance from the RE atom to the capping oxygen along the threefold axis is longer than the others while the situation is reversed for the RE-F distances. The RE atom resides in a site with $\mathrm{C}_{3 \mathrm{v}}$ point symmetry. The four different $\mathrm{Dy}-\mathrm{O}(\mathrm{F})-$ Dy angles are between 102 and $116^{\circ}$.

### 3.2. Analysis of the optical spectra

3.2.1. Selection rules. The electronic transitions between the free ion levels of $\mathrm{Dy}^{3+}$ are induced either by the electric (e.d.) or magnetic dipole (m.d.) interactions and have thus multipolar character. The bulk of the transitions originates from the e.d. interactions according to the $\Delta J \leqslant 6$ selection rule but the probability of observing the m.d. transitions with the selection rule $\Delta J=0, \pm 1$ is non-negligible [29].

For the Kramers ions such as $\mathrm{Dy}^{3+}$, the number of the Stark levels for a particular free ion ${ }^{2 S+1} L_{J}$ state is $J+1 / 2$ for any symmetry lower than cubic. This gives rise to a total of 1001 levels for the $4 f^{9}$ configuration [30]. For the $\mathrm{C}_{3 \mathrm{v}}$ site symmetry, the $\mathrm{Dy}^{3+}$ ion has only two types of Stark level corresponding to the irreducible representation $\mathrm{D}_{1 / 2}$ and the Kramers conjugate level $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$. According to the group theoretical selection rules, the transitions between all c.f. levels are allowed as both the e.d. and m.d. transitions [30].
3.2.2. Luminescence spectra. The visible emission of the $\mathrm{Dy}^{3+}$ ion originating from the ${ }^{4} \mathrm{~F}_{9 / 2}$ state to ${ }^{6} \mathrm{H}_{13 / 2}$ and ${ }^{6} \mathrm{H}_{11 / 2}$ was recorded for the LaOF and GdOF matrices. The emission to the ${ }^{6} \mathrm{H}_{15 / 2}$ ground state could not be observed because the ${ }^{4} \mathrm{~F}_{9 / 2} \rightarrow{ }^{6} \mathrm{H}_{15 / 2}$ transition energy was too close to the $\mathrm{Ar}^{+}$excitation line at 457.9 nm . Not all components of the ${ }^{6} \mathrm{H}_{13 / 2}$ and ${ }^{6} \mathrm{H}_{11 / 2}$ states were observed since only five and four out of seven and six, respectively, were found in the GdOF matrix (table 1). The data for the LaOF matrix were even sparser. The energy levels observed in the GdOF matrix were used in the calculations to minimize the effect of the ionic radius difference between the $\mathrm{Dy}^{3+}$ ion and the host cation.

The c.f. splitting of the ${ }^{6} \mathrm{H}_{11 / 2}$ state was observed from both the luminescence and absorption measurements. The DyOF matrix was used for the absorption measurements and thus the environment of the $\mathrm{Dy}^{3+}$ ion is different from the LaOF and GdOF matrices. Due to the difference in the ionic radius between the $\mathrm{La}^{3+}$ or $\mathrm{Gd}^{3+}$ and the $\mathrm{Dy}^{3+}$ ion, the energies and splitting of the ${ }^{6} \mathrm{H}_{11 / 2}$ Stark levels were not exactly the same. The maximum deviation of the Stark-level energies observed in the luminescence and absorption measurements was about $20 \mathrm{~cm}^{-1}$ because of the different matrices.
3.2.3. Absorption spectra. The optical absorption of $\mathrm{Dy}^{3+}$ in DyOF was measured at selected temperatures between 9 and 300 K in order to find out the splitting of the ${ }^{6} \mathrm{H}_{15 / 2}$ ground state inaccessible by the luminescence measurements. From the hot band absorption, the energies of the six lowest Stark levels of the ground state $(0,12,57,72,134$, and $449 \mathrm{~cm}^{-1}$ ) were confirmed.

In the NIR area, 31 Stark levels out of the theoretical 39 for the ${ }^{6} \mathrm{H}_{11 / 2-5 / 2}$ and ${ }^{6} \mathrm{~F}_{11 / 2-1 / 2}$ states were observed (figures 1 and 2). Five Stark levels out of six were observed for the first excited state ${ }^{6} \mathrm{H}_{11 / 2}$ from absorption spectra between 1700 and 1650 nm . The absorption to the ${ }^{6} \mathrm{H}_{9 / 2}$ and ${ }^{6} \mathrm{~F}_{11 / 2}$ as well as to the ${ }^{6} \mathrm{H}_{7 / 2}$ and ${ }^{6} \mathrm{~F}_{9 / 2}$ states partially overlaps in the energy region from 1220 to 1333 and 1043 to 1120 nm , respectively. The region containing the ${ }^{6} \mathrm{H}_{9 / 2}$ and ${ }^{6} \mathrm{~F}_{11 / 2}$ states was characterized by two large combination lines which are

Optical spectra of DyOF
Table 1. The observed and calculated energy levels of DyOF in $\mathrm{cm}^{-1}$ units.

| The main component |  | $\mathrm{E}_{\text {obs }}$ | $\mathrm{E}_{\text {calc }}$ | The main component |  | $\mathrm{E}_{\text {obs }}$ | $\mathrm{E}_{\text {calc }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{6} \mathrm{H}_{15 / 2}$ 1/2 | $\mathrm{D}_{1 / 2}$ | 0.1 | -5 | ${ }^{6} \mathrm{~F}_{5 / 2}{ }_{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 12613 | 12604 |
| 15/2 | ( $\mathrm{S}_{1}, \mathrm{~S}_{3}$ ) | 12 | 3 | ${ }^{6} \mathrm{~F}_{3 / 2}{ }^{3 / 2}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 13287 | 13296 |
| 13/2 | $\mathrm{D}_{1 / 2}$ | 57 | 64 | 1/2 | $\mathrm{D}_{1 / 2}$ | 13309 | 13304 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 72 | 67 | ${ }^{6} \mathrm{~F}_{1 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 13832 | 13837 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 134 | 130 | ${ }^{4} \mathrm{~F}^{9 / 2} 9 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 20862 | 20881 |
| -11/2 | $\mathrm{D}_{1 / 2}$ | 446 | 424 | 1/2 | $\mathrm{D}_{1 / 2}$ |  | 20911 |
| 7/2 | $\mathrm{D}_{1 / 2}$ |  | 485 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 21052 | 21044 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 492 | -5/2 | $\mathrm{D}_{1 / 2}$ | 21084 | 21072 |
| ${ }^{6} \mathrm{H}_{13 / 2}{ }^{13 / 2}$ | $\mathrm{D}_{1 / 2}$ | 3526 | 3531 | 7/2 | $\mathrm{D}_{1 / 2}$ | 21505 | 21490 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 3547 | 3570 | ${ }^{4}$ I3 ${ }_{15 / 2} \quad 15 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 21816 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 3568 | 3581 | 1/2 | $\mathrm{D}_{1 / 2}$ | 21934 | 21907 |
| 7/2 | $\mathrm{D}_{1 / 2}$ |  | 3587 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 21953 | 21981 |
| 1/2 | $\mathrm{D}_{1 / 2}$ | 3685 | 3711 | -5/2 | $\mathrm{D}_{1 / 2}$ | 22044 | 22008 |
| -11/2 | $\mathrm{D}_{1 / 2}$ | 3743 | 3750 | 1/2 | $\mathrm{D}_{1 / 2}$ | 22261 | 22250 |
| 9/2 | ( $\mathrm{S}_{1}, \mathrm{~S}_{3}$ ) |  | 3766 | -9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 22290 |
| ${ }^{6} \mathrm{H}_{11 / 2}-5 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 5896 | 13/2 | $\mathrm{D}_{1 / 2}$ |  | 22304 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 5929 | 5921 | 11/2 | $\mathrm{D}_{1 / 2}$ | 22320 | 22315 |
| 7/2 | $\mathrm{D}_{1 / 2}$ | 5943 | 5927 | ${ }^{4} \mathrm{G} 4_{11 / 2}{ }^{\text {9/2 }}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 23334 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 5959 | 5973 | 7/2 | $\mathrm{D}_{1 / 2}$ | 23379 | 23343 |
| -11/2 | $\mathrm{D}_{1 / 2}$ | 5977 | 5980 | -5/2 | $\mathrm{D}_{1 / 2}$ | 23390 | 23366 |
| 1/2 | $\mathrm{D}_{1 / 2}$ | 5998 | 6008 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 23459 | 23478 |
| ${ }^{6} \mathrm{H}_{9 / 2}-5 / 2$ | $\mathrm{D}_{1 / 2}$ | 7571 | 7571 | -11/2 | $\mathrm{D}_{1 / 2}$ | 23472 | 23487 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 7622 | 7586 | 1/2 | $\mathrm{D}_{1 / 2}$ | 23535 | 23572 |
| ${ }^{6} \mathrm{~F}_{11 / 2}$ 1/2 | $\mathrm{D}_{1 / 2}$ | 7643 | 7666 | ${ }^{4} \mathrm{M}_{21 / 2}{ }^{21 / 2}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 24730 | 24725 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 7684 | 7672 | 1/2 | $\mathrm{D}_{1 / 2}$ | 24793 | 24787 |
| -11/2 | $\mathrm{D}_{1 / 2}$ | 7712 | 7698 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 24810 | 24820 |
| -5/2 | $\mathrm{D}_{1 / 2}$ |  | 7826 | -5/2 | $\mathrm{D}_{1 / 2}$ | 24831 | 24843 |
| ${ }^{6} \mathrm{H}_{9 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 7855 | 7851 | 1/2 | $\mathrm{D}_{1 / 2}$ | 24970 | 24994 |
| ${ }^{6} \mathrm{~F}_{11 / 2-5 / 2}$ | $\mathrm{D}_{1 / 2}$ | 7870 | 7860 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 25106 | 25107 |
| ${ }^{6} \mathrm{H}_{9 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 7890 | 7895 | 19/2 | $\mathrm{D}_{1 / 2}$ | 25118 | 25125 |
| ${ }^{6} \mathrm{~F}_{11 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 8113 | -17/2 | $\mathrm{D}_{1 / 2}$ |  | 25166 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 8164 | 8167 | 13/2 | $\mathrm{D}_{1 / 2}$ | 25307 | 25305 |
| ${ }^{6} \mathrm{~F}_{9 / 2}-5 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 8931 | -11/2 | $\mathrm{D}_{1 / 2}$ |  | 25330 |
| ${ }^{6} \mathrm{H}_{7 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 8988 | 8982 | 15/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 25331 |
| ${ }^{6} \mathrm{~F}_{9 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ | 9008 | 8993 | 13/2 | $\mathrm{D}_{1 / 2}$ | 25362 | 25347 |
| 1/2 | $\mathrm{D}_{1 / 2}$ |  | 9219 | ${ }^{4} \mathrm{I}_{13 / 2} \quad 13 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 25406 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 9240 | 9220 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 25435 | 25427 |
| ${ }^{6} \mathrm{H}_{7 / 2}{ }^{\text {1/2 }}$ | $\mathrm{D}_{1 / 2}$ | 9252 | 9260 | 1/2 | $\mathrm{D}_{1 / 2}$ | 25449 | 25436 |
| ${ }^{6} \mathrm{~F}_{9 / 2} 9 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 9265 | ${ }^{4} \mathrm{~K} 1_{17 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 25626 | 25618 |
| ${ }^{6} \mathrm{H}_{7 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ | 9295 | 9317 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 25653 |
|  | $\mathrm{D}_{1 / 2}$ | 9575 | 9588 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 25653 |
| ${ }^{6} \mathrm{H}_{5 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 10193 | -17/2 | $\mathrm{D}_{1 / 2}$ | 25687 | 25688 |
| -5/2 | $\mathrm{D}_{1 / 2}$ |  | 10303 | -11/2 | $\mathrm{D}_{1 / 2}$ | 25731 | 25780 |
| $3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 10348 | 10335 | ${ }^{4} \mathrm{~F} 3_{7 / 2} 1 / 2$ | $\mathrm{D}_{1 / 2}$ | 25835 | 25822 |
| ${ }^{6} \mathrm{~F}_{7 / 2-5 / 2}$ | $\mathrm{D}_{1 / 2}$ | 10995 | 11004 | ${ }^{4} \mathrm{~K} 1_{17 / 2}{ }^{\text {9/2 }}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 25843 | 25842 |
| 1/2 | $\mathrm{D}_{1 / 2}$ | 11170 | 11177 | 1/2 | $\mathrm{D}_{1 / 2}$ | 25860 | 25880 |
| 7/2 | $\mathrm{D}_{1 / 2}$ | 11211 | 11209 | ${ }^{4} \mathrm{~F} 3_{7 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 25878 | 25891 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 11220 | 11220 | ${ }^{4} \mathrm{~K} 1_{17 / 2}-5 / 2$ | $\mathrm{D}_{1 / 2}$ | 25917 | 25915 |
| ${ }^{6} \mathrm{~F}_{5 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 12446 | 12457 | ${ }^{4} \mathrm{I}_{13 / 2} 1 / 2$ | $\mathrm{D}_{1 / 2}$ | 25931 | 25955 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 12456 | 12473 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 25956 |

Table 1. (Continued)

| The main component |  | $\mathrm{E}_{\text {obs }}$ | $\mathrm{E}_{\text {calc }}$ | The main component |  | $\mathrm{E}_{\text {obs }}$ | $\mathrm{E}_{\text {calc }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{4} \mathrm{I}_{13 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ | 25977 | 25986 | ${ }^{4} \mathrm{M}_{17 / 2-17 / 2}$ | $\mathrm{D}_{1 / 2}$ | 29891 | 29902 |
| ${ }^{4} \mathrm{~K} 1_{17 / 2} 13 / 2$ | $\mathrm{D}_{1 / 2}$ | 25989 | 26001 | 1/2 | $\mathrm{D}_{1 / 2}$ | 29926 | 29916 |
| ${ }^{4} \mathrm{I}_{13 / 2} 9 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 26010 | -11/2 | $\mathrm{D}_{1 / 2}$ | 29961 | 29951 |
| -11/2 | $\mathrm{D}_{1 / 2}$ | 26033 | 26036 | 13/2 | $\mathrm{D}_{1 / 2}$ | 29976 | 29965 |
| ${ }^{4} \mathrm{M}_{19 / 2}{ }^{19 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 26073 | 15/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 29996 | 29967 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 26107 | ${ }^{4} \mathrm{G}_{4}{ }_{9 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 30168 |
| -5/2 | $\mathrm{D}_{1 / 2}$ |  | 26109 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 30181 |
| 7/2 | $\mathrm{D}_{1 / 2}$ | 26162 | 26133 | ${ }^{4} \mathrm{M}_{17 / 2} \quad 13 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 30242 |
| 13/2 | $\mathrm{D}_{1 / 2}$ | 26308 | 26315 | -11/2 | $\mathrm{D}_{1 / 2}$ |  | 30304 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 26376 | 26383 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 30343 | 30314 |
| 13/2 | $\mathrm{D}_{1 / 2}$ |  | 26388 | ${ }^{6} \mathrm{P}_{3 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 30754 |
| 1/2 | $\mathrm{D}_{1 / 2}$ |  | 26411 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 30777 | 30765 |
| -17/2 | $\mathrm{D}_{1 / 2}$ | 26442 | 26434 | ${ }^{4} \mathrm{~K} 1_{15 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 30817 | 30819 |
| 15/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 26503 | 26463 | 13/2 | $\mathrm{D}_{1 / 2}$ | 30900 | 30904 |
| ${ }^{4} \mathrm{P} 2_{3 / 2}$ 1/2 | $\mathrm{D}_{1 / 2}$ | 27316 | 27311 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 30918 | 30910 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 27333 | 27333 | 11/2 | $\mathrm{D}_{1 / 2}$ | 30983 | 30976 |
| ${ }^{6} \mathrm{P}_{5 / 2-5 / 2}$ | $\mathrm{D}_{1 / 2}$ | 27380 | 27381 | 15/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 31007 | 30981 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 27385 | 27383 | -5/2 | $\mathrm{D}_{1 / 2}$ | 31035 | 31032 |
| 1/2 | $\mathrm{D}_{1 / 2}$ | 27395 | 27405 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 31224 |
| ${ }^{4} \mathrm{I} 3_{11 / 2-11 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 27723 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 31235 | 31240 |
| 1/2 | $\mathrm{D}_{1 / 2}$ | 27785 | 27810 | ${ }^{4} \mathrm{~L}_{19 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ | 31256 | 31261 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 27797 | 27812 | -17/2 | $\mathrm{D}_{1 / 2}$ | 31272 | 31283 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 27856 | 27856 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 31284 | 31311 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 28010 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 31314 |
| 7/2 | $\mathrm{D}_{1 / 2}$ | 28036 | 28016 | -11/2 | $\mathrm{D}_{1 / 2}$ |  | 31351 |
| ${ }^{4} \mathrm{M}_{15 / 2}{ }^{15 / 2}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 28172 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 31359 |
| 7/2 | $\mathrm{D}_{1 / 2}$ |  | 28232 | 15/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 31392 | 31386 |
| -5/2 | $\mathrm{D}_{1 / 2}$ |  | 28236 | 1/2 | $\mathrm{D}_{1 / 2}$ | 31420 | 31423 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 28249 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 31454 |
| ${ }^{6} \mathrm{P}_{7 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 28390 | 28390 | 13/2 | $\mathrm{D}_{1 / 2}$ | 31482 | 31462 |
| 1/2 | $\mathrm{D}_{1 / 2}$ |  | 28391 | ${ }^{4} \mathrm{~K} 1_{15 / 2}$-11/2 | $\mathrm{D}_{1 / 2}$ | 31521 | 31544 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 28407 | 28404 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 31594 |
| 7/2 | $\mathrm{D}_{1 / 2}$ | 28432 | 28432 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 31620 |
| ${ }^{4} \mathrm{M}_{15 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 28628 | 28616 | ${ }^{4} \mathrm{G} 2_{7 / 2}{ }_{7 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 31714 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 28643 | 28631 | ${ }^{4} \mathrm{D} 2_{5 / 2} 1 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 31867 |
| 13/2 | $\mathrm{D}_{1 / 2}$ | 28657 | 28674 | ${ }^{4} \mathrm{P} 2_{1 / 2}{ }_{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 32013 |
| -11/2 | $\mathrm{D}_{1 / 2}$ | 28725 | 28741 | ${ }^{4} \mathrm{D} 2_{5 / 2} 5 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 32039 |
| ${ }^{4} \mathrm{I}_{3 / 2}{ }^{\text {9/2 }}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 29336 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 32095 |
| 1/2 | $\mathrm{D}_{1 / 2}$ |  | 29373 | ${ }^{4} \mathrm{H}_{13 / 2}{ }^{7 / 2}$ | $\mathrm{D}_{1 / 2}$ | 32956 | 32987 |
| 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 29493 | -11/2 | $\mathrm{D}_{1 / 2}$ | 33021 | 33038 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 29510 | 29496 | 13/2 | $\mathrm{D}_{1 / 2}$ | 33042 | 33040 |
| ${ }^{4} \mathrm{G} 4_{9 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ | 29640 | 29635 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 33052 | 33041 |
| ${ }^{4} \mathrm{~F} 3_{5 / 2} 1 / 2$ | $\mathrm{D}_{1 / 2}$ | 29655 | 29659 | ${ }^{4} \mathrm{~K} 1_{13 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 33063 | 33069 |
| ${ }^{4} \mathrm{I}_{9 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 29811 | ${ }^{4} \mathrm{H}_{13 / 2}{ }^{\text {9/2 }}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 33083 | 33079 |
| ${ }^{4} \mathrm{M}_{17 / 2-5 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 29816 | ${ }^{4} \mathrm{~K} 1_{13 / 2}-11 / 2$ | $\mathrm{D}_{1 / 2}$ | 33100 | 33083 |
| ${ }^{4} \mathrm{I}_{9 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 29825 | ${ }^{4} \mathrm{H} 1_{13 / 2}-11 / 2$ | $\mathrm{D}_{1 / 2}$ | 33329 | 33333 |
| ${ }^{4} \mathrm{M}_{17 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 29847 | 29842 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 33346 | 33334 |
| ${ }^{4} \mathrm{I}_{9 / 2}{ }^{\text {-5/2 }}$ | $\mathrm{D}_{1 / 2}$ | 29865 | 29870 | ${ }^{4} \mathrm{~K} 1_{13 / 2}{ }^{\text {9/2 }}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 33350 | 33352 |
| ${ }^{4} \mathrm{M}_{17 / 2} 9 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 29875 | 29888 | ${ }^{4} \mathrm{H} 1_{13 / 2}{ }^{13 / 2}$ | $\mathrm{D}_{1 / 2}$ | 33376 | 33365 |

Table 1. (Continued)

| The main component |  | $\mathrm{E}_{\text {obs }}$ | $\mathrm{E}_{\text {calc }}$ | The main component |  | $\mathrm{E}_{\text {obs }}$ | $\mathrm{E}_{\text {calc }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{4} \mathrm{H} 1_{13 / 2}{ }^{13 / 2}$ | $\mathrm{D}_{1 / 2}$ | 33401 | 33409 | ${ }^{4} L_{17 / 2}-5 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 34301 |
| 1/2 | $\mathrm{D}_{1 / 2}$ |  | 33419 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 34345 |
| ${ }^{4} \mathrm{~L}_{13 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ | 33479 | 33449 | 17/2 | $\mathrm{D}_{1 / 2}$ | 34349 | 34348 |
| ${ }^{4} \mathrm{~F}_{3 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ | 33519 | 33522 | 1/2 | $\mathrm{D}_{1 / 2}$ |  | 34368 |
| 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 33526 | 9/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 34379 | 34378 |
| ${ }^{4} \mathrm{H} 1_{11 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 33823 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 34396 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 33843 | 33842 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 34403 |
| ${ }^{4} \mathrm{D} 3_{7 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 33853 | 33846 | ${ }^{4} \mathrm{~F}_{5 / 2}{ }_{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 34524 |
| 7/2 | $\mathrm{D}_{1 / 2}$ | 33863 | 33858 | ${ }^{4} \mathrm{G} 2_{11 / 2} 7 / 2$ | $\mathrm{D}_{1 / 2}$ | 34679 | 34683 |
| ${ }^{4} \mathrm{H}_{11 / 2}{ }_{9 / 2}$ | ( $\mathrm{S}_{1}, \mathrm{~S}_{3}$ ) |  | 33938 | 9/2 | ( $\mathrm{S}_{1}, \mathrm{~S}_{3}$ ) |  | 34683 |
| 1/2 | $\mathrm{D}_{1 / 2}$ |  | 33948 | 5/2 | $\mathrm{D}_{1 / 2}$ | 34719 | 34724 |
| -5/2 | $\mathrm{D}_{1 / 2}$ | 34013 | 34031 | 3/2 | $\left(S_{1}, S_{3}\right)$ |  | 34836 |
| 7/2 | $\mathrm{D}_{1 / 2}$ |  | 34077 | 11/2 | $\mathrm{D}_{1 / 2}$ | 34863 | 34854 |
| ${ }^{4} \mathrm{H} 3_{11 / 2}{ }^{3 / 2}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 34090 | 11/2 | $\mathrm{D}_{1 / 2}$ |  | 34925 |
| ${ }^{4} \mathrm{H}_{11 / 2}{ }^{3 / 2}$ | ( $\mathrm{S}_{1}, \mathrm{~S}_{3}$ ) |  | 34105 | ${ }^{4} \mathrm{~K} 1_{11 / 2}{ }_{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 35632 |
| 7/2 | $\mathrm{D}_{1 / 2}$ |  | 34112 | 9/2 | ( $\mathrm{S}_{1}, \mathrm{~S}_{3}$ ) |  | 35657 |
| ${ }^{4} \mathrm{G1}_{9 / 2}{ }_{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 34125 | -11/2 | $\mathrm{D}_{1 / 2}$ |  | 35729 |
| ${ }^{4} \mathrm{H} 1_{11 / 2}{ }^{\text {9/2 }}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 34152 | $-11 / 2$ | $\mathrm{D}_{1 / 2}$ |  | 35753 |
| 1/2 | $\mathrm{D}_{1 / 2}$ |  | 34157 | ${ }^{4} \mathrm{H}_{17 / 2} 3 / 2$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 35755 |
| 5/2 | $\mathrm{D}_{1 / 2}$ | 34204 | 34201 | ${ }^{4} \mathrm{~K} 1_{11 / 2}{ }^{1 / 2}$ | $\mathrm{D}_{1 / 2}$ |  | 35772 |
| ${ }^{4} \mathrm{~L}_{17 / 2}{ }^{\text {15/2 }}$ | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ | 34223 | 34223 | 3/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 35891 |
| 1/2 | $\mathrm{D}_{1 / 2}$ | 34272 | 34253 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 35915 |
| 15/2 | $\left(\mathrm{S}_{1}, \mathrm{~S}_{3}\right)$ |  | 34291 | 7/2 | $\mathrm{D}_{1 / 2}$ |  | 35951 |
| 13/2 | $\mathrm{D}_{1 / 2}$ |  | 34301 | -5/2 | $\mathrm{D}_{1 / 2}$ |  | 36045 |

due to the overlap of three Stark levels. Absorption to nine of the 11 Stark levels of the ${ }^{6} \mathrm{H}_{9 / 2}$ and ${ }^{6} \mathrm{~F}_{11 / 2}$ states was observed. The ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{~F}_{11 / 2}$ transition is a hypersensitive transition with intensity particularly sensitive to the host material [31,32]. The intensity of this transition is lower for DyOCl [33] than for DyOF when compared to the intensities of the other transitions. The ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{H}_{7 / 2}$ and ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{~F}_{9 / 2}$ transitions have low intensity when compared to the ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{~F}_{11 / 2}$ transition. From the ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{H}_{7 / 2},{ }^{6} \mathrm{~F}_{9 / 2}$ transitions six out of a theoretical nine Stark levels were observed.

The intensities of the ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{H}_{5 / 2}$ and ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{~F}_{1 / 2}$ transitions were low. The transition to the ${ }^{6} \mathrm{~F}_{1 / 2}$ state is forbidden due to the $\Delta J \leqslant 6$ free ion selection rule [29] but was observed anyhow owing to the breakdown of these selection rules by the c.f. effect. The poor quality of the spectra in the region between 800 and 950 nm (the ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{~F}_{7 / 2}$ transition) was due to the detector change.

In the visible and UV regions, the density of the absorption lines increases (figure 3). The ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{4} \mathrm{I}_{15 / 2}$ and ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{4} \mathrm{G} 4_{15 / 2}$ transitions at wavelengths from 448 to 458 and 424 to 428 nm , respectively, yielded the last well resolved lines, beyond which the $J$ mixing of states becomes high. The transition of high intensity at 352 nm was assigned to ${ }^{6} \mathrm{H}_{15 / 2} \rightarrow{ }^{6} \mathrm{P}_{7 / 2}$. The last Stark levels, attributed to the absorption to the ${ }^{4} \mathrm{G} 2_{11 / 2}$ state, were observed at 287 nm . The transitions to the higher levels of the $4 f^{9}$ configuration were hidden by a large band absorption due to the activator-host lattice interaction. Most of the levels of the $4 f^{9}$ configuration are thus inaccessible with one-photon absorption techniques.

The lines marked with an asterisk in the absorption spectra are associated with transitions which originate from the coupling between the pure electronic zero-phonon lines and lattice vibrations [34]. The coupling is most clearly seen in the NIR region (figures 1 and 2). The


Figure 1. The NIR absorption spectra of DyOF at 10 K at wavelengths between 1000 and 1700 nm . The vibronic transitions are marked with an asterisk.


Figure 2. The NIR absorption spectra of DyOF at 10 K at wavelengths between 720 and 975 nm . The vibronic transitions are marked with an asterisk.
vibronic lines could be confirmed by the Raman and IR spectra of the REOF series [35]. Lines corresponding to frequencies 250 and $360 \mathrm{~cm}^{-1}$ were usually found. Moreover, the analysis of the line shapes and the line intensities as a function of the temperature excluded the vibronic lines with high probability.

The number of the Stark levels observed from absorption spectra was 149 which represents 39 SLJ manifolds including the ${ }^{6} \mathrm{H}_{15 / 2}$ ground state as well as ${ }^{6} \mathrm{H}_{11 / 5-5 / 2}$, ${ }^{6} \mathrm{~F}_{11 / 2-1 / 2},{ }^{4} \mathrm{~F} 3_{9 / 2-3 / 2},{ }^{4} \mathrm{I} 3_{15 / 2-9 / 2},{ }^{4} \mathrm{G} 4_{11 / 2-7 / 2},{ }^{4} \mathrm{M} 2_{1 / 2-17 / 2},{ }^{4} \mathrm{~K} 1_{17 / 2,11 / 2},{ }^{6} \mathrm{P} 3_{7 / 2-3 / 2},{ }^{4} \mathrm{D} 2_{5 / 2}$, ${ }^{4} \mathrm{P} 2_{3 / 2},{ }^{4} \mathrm{~L}_{13 / 2,11 / 2},{ }^{4} \mathrm{H} 1_{13 / 2-11 / 2},{ }^{4} \mathrm{D} 3_{11 / 2},{ }^{4} \mathrm{G} 1_{11 / 2}$, and ${ }^{4} \mathrm{G} 2_{11 / 2}$ (table 1).

### 3.3. Simulation of the energy level scheme

3.3.1. Theoretical treatment of experimental data. The most important interactions describing the free ion energy level structure of the $\mathrm{RE}^{3+}$ ions are the Coulombic and


Figure 3. The UV and visible absorption spectra of DyOF at 10 K at wavelengths between 275 and 485 nm . The vibronic transitions are marked with an asterisk.
spin-orbit interactions. When only these interactions are used, however, discrepancies of an order of a few hundred wave numbers remain between the calculated and experimental energy levels [36]. Minor contributions include the spin-independent two- and three-body interactions between the 4 f and different excited configurations as well as the c.f. effect. The corresponding effective Hamiltonian $H$ can then be described as follows [29]:

$$
\begin{align*}
H=H_{0}- & \sum_{k=0,1,2,3} E_{k}(n \mathrm{f}, n \mathrm{f}) \mathrm{e}^{k}+\zeta_{4 f} A_{\mathrm{SO}}+\alpha L(L+1)+\beta G\left(\mathrm{G}_{2}\right)+\gamma \mathrm{G}\left(\mathrm{R}_{7}\right) \\
& +\sum_{k=2,3,4,6,7,8} T^{k} t_{k}+\sum_{k, q, i} B_{q}^{k} C_{q}^{k}(i) \tag{1}
\end{align*}
$$

where $H_{0}$ is the one-electron part of the free ion Hamiltonian where the potential seen by an electron on a rare earth ion is treated as spherically symmetric. $E_{k}$ (Racah parameters) and $\zeta_{4 \mathrm{f}}$ are the electrostatic and spin-orbit integrals whereas $\mathrm{e}^{k}$ and $A_{\text {SO }}$ represent the angular parts of these interactions, respectively. Spin-independent configuration interaction arises from coupling of configurations by the Coulomb interaction and can be described by the two- and three-body terms. The former include the Trees parameters $\alpha, \beta$, and $\gamma$ with $L$ as the total orbital angular momentum of the levels involved. $G\left(\mathrm{G}_{2}\right)$ and $G\left(\mathrm{R}_{7}\right)$ are the Casimir operators for the Lie groups $\mathrm{G}_{2}$ and $\mathrm{R}_{7}$, respectively. The three-body interactions are parametrized with Judd parameters, $T^{k}$, and the corresponding effective operators $t_{k}$ ( $k=2,3,4,6,7$, and 8 ) which transform according to definite irreducible representations of $\mathrm{G}_{2}$ and $\mathrm{R}_{7}$ [29].

The standard, one-electron c.f. Hamiltonian $H_{C F}$ is described by the product $B_{q}^{k}(i) C_{q}^{k}(i)$ with summation $i$ over all electrons of the ion of interest [29]. $B_{q}^{k}$ are the real c.f. parameters and $C_{q}^{k}$ the spherical tensors of rank $k$ equal to or less than six for the 4 f elements. The values of $k$ and $q$ will be constrained by the point symmetry of the $\mathrm{RE}^{3+}$ site. For the $\mathrm{C}_{3 \mathrm{v}}$ site symmetry, the c.f. Hamiltonian has the following form:
$H_{\mathrm{CF}}=B_{0}^{2} C_{0}^{2}+B_{0}^{4} C_{0}^{4}+B_{3}^{4}\left(C_{-3}^{4}-C_{3}^{4}\right)+B_{0}^{6} C_{0}^{6}+B_{3}^{6}\left(C_{-3}^{6}-C_{3}^{6}\right)+B_{6}^{6}\left(C_{-6}^{6}+C_{6}^{6}\right)$
The best fit set of the c.f. parameters was obtained by the least-squares refinement through
minimizing the rms function $\sigma$ between the observed and calculated energy level values [37].

As the starting values for the free ion parameters those for the $\mathrm{LaF}_{3}: \mathrm{RE}^{3+}$ system [1] were utilized. The c.f. parameters from earlier calculations for the other REOF:RE ${ }^{3+}$ systems [20-24] were employed as the a priori values, too. The method used varies simultaneously both the free ion and c.f. parameters using a non-truncated set of wave functions. Calculations involve a diagonalization of two square matrices of sizes 666 and 670 [38]. As the literature survey indicated, the two requirements for a complete simulation, i.e. the use of a complete set of the basis functions and the simultaneous treatment of the free ion and c.f. effects, were fulfilled in this work for the first time.
3.3.2. Analysis of experimental data. The energy level scheme of the $\mathrm{Dy}^{3+}$ ion was resolved according to the group theoretical rules for the $\mathrm{C}_{3 \mathrm{v}}$ site symmetry. 153 Stark levels out of 1001 levels were derived from the absorption and luminescence spectra. A set of 20 parameters including 14 free ion and six c.f. parameters was used in the final simulation of the energy level scheme. The degrees of freedom are high enough to allow a reliable variation of all parameters involved.

In the beginning of the simulation only the Racah parameter $E_{0}$ and the spin-orbit coupling constant $\zeta_{4 \mathrm{f}}$ together with the c.f. parameters were refined by using the c.f. components of the two lowest terms ${ }^{6} \mathrm{H}$ and ${ }^{6} \mathrm{~F}$. After achieving constant values of the c.f. parameters more energy levels and additional free ion parameters such as the other Racah parameters, $E_{1}, E_{2}$, and $E_{3}$, and those of Trees, $\alpha, \beta$, and $\gamma$, were included into the simulations. The Judd parameters were the last ones incorporated. However, the parameters $T^{3}$ and $T^{4}$ were left with fixed values after a few iteration cycles because they assumed unstable values.

The energy level scheme of the $\mathrm{Dy}^{3+}$ ion was well simulated as a whole. A good agreement between the observed and calculated level values was achieved with the rms deviation of $17 \mathrm{~cm}^{-1}$. The isolated ${ }^{6} \mathrm{H}$ and ${ }^{6} \mathrm{~F}$ terms of the $\mathrm{Dy}^{3+}$ ion are well separated from the higher excited terms and can be treated as a separate system which gives a very good fit. The inclusion of the higher excited levels with increasing mixing of the $J$ multiplets changes the best fit $B_{q}^{k}$ sets and impairs somewhat the overall simulation. The model involving the adjustment of both the free ion and c.f. parameters usually gives a good fit. Certain 'anomalous' multiplets which remain badly fitted, such as ${ }^{1} D_{2}$ of $\operatorname{Pr}^{3+}[39],{ }^{2} \mathrm{H} 2_{11 / 2}$ of $\mathrm{Nd}^{3+}$ [40], or ${ }^{3} \mathrm{~K}_{8}$ of $\mathrm{Ho}^{3+}$ [41], are found when the one-electron c.f. parameters are optimized alone [46]. In the case of DyOF no anomalous simulation was obtained for any individual state.
3.3.3. Free ion parametrization. The calculated free ion parameters (table 2) are similar to those obtained for $\mathrm{LaF}_{3}: \mathrm{RE}^{3+}[1]$ or $\mathrm{LaCl}_{3}: \mathrm{RE}^{3+}$ [4]. This was to be expected since the energies of the centres of the gravity of the c.f. multiplets, i.e. the ${ }^{2 S+1} L_{J}$ states, do not vary much from one host lattice to another [31]. In contrast, the Racah parameters $E_{0}, E_{1}, E_{2}$, and $E_{3}$, and the spin-orbit coupling constant $\zeta_{4 f}\left(23429,4689,23.17,478\right.$, and $870 \mathrm{~cm}^{-1}$ for $\mathrm{Nd}^{3+}$ [21] and $46314,5204,25.46,518$, and $1150 \mathrm{~cm}^{-1}$ for $\mathrm{Sm}^{3+}$ [22]) increase with increasing number of 4 f electrons in the REOF series. The interconfigurational terms, i.e. the Trees and Judd parameters, show no clear trends. The parameters are well defined with low estimated standard deviations indicating the high quality of the simulation.

Table 2. The phenomenological free ion and c.f. parameters with estimated standard deviation. The calculated c.f. parameters were obtained by a modified point charge model.

| Parameter | Exp. $\left(\mathrm{cm}^{-1}\right)$ | Calc. $\left(\mathrm{cm}^{-1}\right)$ |
| :--- | :--- | :--- |
| $E_{0}$ | $55299(1)$ |  |
| $E_{1}$ | $6171.9(2)$ |  |
| $E_{2}$ | $30.40(1)$ |  |
| $E_{3}$ | $623.84(5)$ |  |
| $\alpha$ | $18.11(2)$ |  |
| $\beta$ | $-603(2)$ |  |
| $\gamma$ | $1600(1)$ |  |
| $T^{2}$ | $350(1)$ |  |
| $T^{3}$ | $[78]$ |  |
| $T^{4}$ | $[41]$ |  |
| $T^{6}$ | $-360(11)$ |  |
| $T^{7}$ | $350(8)$ |  |
| $T^{8}$ | $345(6)$ |  |
| $\zeta_{4 f}$ | $1915(1)$ |  |
| $B_{0}^{2}$ | $57(16)$ | -870 |
| $B_{0}^{4}$ | $1717(31)$ | 1831 |
| $B_{0}^{4} / B_{3}^{4}$ | -1.350 | -1.167 |
| $B_{3}^{4}$ | $-1272(21)$ | -1569 |
| $B_{0}^{6}$ | $1204(35)$ | 957 |
| $B_{0}^{6} / B_{3}^{6}$ | 2.165 | 1.018 |
| $B_{3}^{6}$ | $556(31)$ | 940 |
| $B_{0}^{6} / B_{6}^{6}$ | 1.881 | 0.865 |
| $B_{0}^{6}$ | $640(31)$ | 1106 |
| No of levels | $153 / 1001$ |  |
| $\sigma$ | 17 |  |

3.3.4. Crystal field parametrization. The present simulation can be considered good with a low rms deviation and the absence of large individual discrepancies between the calculated and experimental data. The $B_{0}^{2}$ value was found to be close to zero whereas the fourth- and sixth-rank parameters assumed high values (table 2) which are approximately restricted to the ratios allowed for the ideal cubic symmetry, i.e. $B_{0}^{4} / B_{3}^{4}=-0.837, B_{0}^{6} / B_{3}^{6}=1.656$, and $B_{0}^{6} / B_{6}^{6}=1.579$ [43]. These results indicate only a slight distortion from cubic structure.

The set of parameters is consistent with those for the REOF: $\mathrm{RE}^{3+}$ cases studied so far (table 3) [20-24]. It is expected that the c.f. effect should decrease as a function of increasing number of 4 f electrons due to the increasing nuclear charge. However, only the $\mathrm{Nd}^{3+}, \mathrm{Sm}^{3+}$, and $\mathrm{Dy}^{3+}$ ions are in their 'natural' environment and for the $\mathrm{Eu}^{3+}$ and $\mathrm{Tb}^{3+}$ ions only the ground term ${ }^{7} \mathrm{~F}_{\mathrm{J}}$ has been employed in the simulation [23, 24]. These facts may affect the comparison. The expected evolution was observed from $\mathrm{Pr}^{3+}$ to $\mathrm{Tb}^{3+}$, but beyond $\mathrm{Tb}^{3+}$ the c.f. effect seems to get stronger again. This can most easily be seen from the evolution of the c.f. strength parameter $S$ (table 3) which is a quantitative measure of the strength of the c.f. interaction in a particular host [7]:

$$
\begin{equation*}
S=\left\{\frac{1}{3} \sum_{k} \frac{1}{2 k+1}\left[\left(B_{0}^{k}\right)^{2}+2 \sum_{q>0}\left(\left(B_{q}^{k}\right)^{2}+\left(S_{q}^{k}\right)^{2}\right)\right]\right\}^{1 / 2} \tag{3}
\end{equation*}
$$

where $B_{q}^{k}$ and $S_{q}^{k}$ are the real and imaginary c.f. parameters.
The influence of the nuclear charge beyond $\mathrm{Tb}^{3+}$ may be cancelled by two effects.

Table 3. The evolution of the free ion and $B_{q}^{k}$ parameters in the REOF series. The data were obtained for $\mathrm{Pr}^{3+}, \mathrm{Nd}^{3+}, \mathrm{Sm}^{3+}, \mathrm{Eu}^{3+}, \mathrm{Tb}^{3+}$, and $\mathrm{Dy}^{3+}$ in YOF [20], NdOF [21], SmOF [22], LaOF [23], LaOF [24], and DyOF (this work) matrices, respectively. All values are in $\mathrm{cm}^{-1}$ units.

| Parameter | YOF:Pr ${ }^{3+}$ | NdOF | SmOF | $\mathrm{LaOF}^{2}: \mathrm{Eu}^{3+}$ | ${\mathrm{LaOF}: \mathrm{Tb}^{3+}}^{\text {Lnits. }}$ | DyOF |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $B_{0}^{2}$ | -124 | -144 | -200 | -28 | -36 | 57 |
| $B_{0}^{4}$ | 1612 | 1839 | 1786 | 1162 | 1344 | 1717 |
| $B_{0}^{4} / B_{3}^{4}$ a | -0.71 | -1.12 | -1.35 | -0.86 | -1.06 | -1.35 |
| $B_{3}^{4}$ | -2276 | -1643 | -1308 | -1347 | -1272 | -1272 |
| $B_{0}^{6}$ | 1237 | 1005 | 1139 | 866 | 835 | 1204 |
| $B_{0}^{6} / B_{3}^{6}$ a | 1.13 | 1.16 | 1.54 | 1.8 | 2.0 | 2.16 |
| $B_{3}^{6}$ | 1096 | 866 | 739 | 482 | 418 | 556 |
| $B_{0}^{6} / B_{6}^{6}$ a | 1.31 | 1.24 | 1.41 | 1.242 | 2.34 | 1.88 |
| $B_{6}^{6}$ | 745 | 812 | 810 | 697 | 357 | 640 |
| $S$ | 780 | 650 | 583 | 490 | 469 | 543 |
| No of levels | $58 / 91$ | $122 / 182^{\mathrm{b}}$ | $195 / 1001^{\mathrm{b}}$ | $21 / 49$ | $19 / 49$ | $153 / 1001^{\mathrm{b}}$ |
| $\sigma$ | 15 | 17 | 17 | 10 | 5 | 17 |

${ }^{\text {a }}$ Ideal cubic ratios: $B_{0}^{4} / B_{3}^{4}=-0.837, B_{0}^{6} / B_{3}^{6}=1.656$, and $B_{0}^{6} / B_{6}^{6}=1.579$ [43].
${ }^{\mathrm{b}}$ Kramers doublets.

Firstly, it was observed that the values of the $A_{q}^{4}$ lattice sum parameters given by equation (4) (next section) remain nearly constant throughout the whole REOF series. In contrast, the $A_{q}^{6}$ parameters increase significantly ( $\sim 20 \%$ ) with decreasing ionic radius from the $\mathrm{Pr}^{3+}$ to the $\mathrm{Dy}^{3+}$ ion. The $\left\langle r^{k}\right\rangle$ free ion radial integrals drastically decrease from $\mathrm{Pr}^{3+}$ to the middle of the $\mathrm{RE}^{3+}$ series but then smoothen out [47]. The evolution of the lattice sum parameters and radial integrals may be the reason for the increase in the c.f. effect beyond $\mathrm{Tb}^{3+}$. Secondly, the higher ( 5 s and 5 d ) orbitals introduce different parity contributions to the 4 f configuration which strengthens the c.f. effect. This effect is expected to become important for the heavier rare earths due to the lanthanide contraction. At the moment, no further conclusions can be drawn because of the lack of information about the effective interactions and the energy separation between the different configurations.

The contributions from the orbitally [44] and/or spin [45] correlated two-electron c.f. interactions may have to be taken into account in the calculations, too. From a theoretical point of view, the additional parameters may give some advantages, but, in practice, their large number requires the use of simplifications and rejection of the less important parameters [46]. The decision to include additional parameters may prove to be rather complicated if the choice is based on the rather inaccurate experimental data. Some success-such as the correct simulation of the ${ }^{2} \mathrm{H} 2_{11 / 2}$ level splitting for the $\mathrm{Nd}^{3+}$ ion [42]-has been achieved by including the two-electron c.f. interaction.

### 3.4. Electrostatic point charge model calculations

The c.f. Hamiltonian is expressed as a sum of products of the Racah tensor operators, $C_{q}^{k}$, and the c.f. parameters, $B_{q}^{k}$ (2). According to the modified electrostatic point charge model (PCEM), the c.f. parameters can also be given as follows:

$$
\begin{equation*}
B_{q}^{k}=\tau^{-k}\left(1-\sigma_{k}\right) A_{q}^{k}\left\langle r^{k}\right\rangle \tag{4}
\end{equation*}
$$

where $\left\langle r^{k}\right\rangle$ are the free ion radial integrals and $A_{q}^{k}$ the lattice sum parameters. The correction factors $\sigma_{k}$ describe the shielding effect of the $6 \mathrm{~s}, 5 \mathrm{~d}$, and 5 p electrons. The expansion factor $\tau$
describes the expansion of the 4 f wave functions in the solid state when compared to the free ion. The expansion factor and the shielding factor $\sigma_{2}$ were calculated as follows [47]: $\tau=0.767-(0.00896 N)=0.6807$ and $\sigma_{2}=0.6902-(0.0121 N)=0.580$ where $N$ is the number of 4 f electrons, nine for the $\mathrm{Dy}^{3+}$ ion. The $\sigma_{4}$ and $\sigma_{6}$ values were assumed as constants across the RE series [47]. The free ion radial integrals $\left\langle r^{k}\right\rangle$ for the $\mathrm{Dy}^{3+}$ ion, $\left\langle r^{2}\right\rangle=0.7814,\left\langle r^{4}\right\rangle=1.505$, and $\left\langle r^{6}\right\rangle=6.148$, have been calculated by the Dirac-Fock method [48].

The electrostatic lattice sum parameters $A_{q}^{k}$ were calculated according to the following formula [49]:

$$
\begin{equation*}
A_{q}^{k}=\mathrm{e}^{2}\left[\frac{(k+q)!}{(k-q)!}\right] \sum_{j} \frac{g_{j}}{\rho_{j}^{k+1}} \frac{\left(1-x_{j}^{2}\right)^{-q / 2}}{2^{k} k!} \frac{\mathrm{d}^{k-q}\left(x_{j}^{2}-1\right)^{2}}{\mathrm{~d} x_{j}^{k-q}} \exp \left(-\mathrm{i} q \beta_{j}\right) \tag{5}
\end{equation*}
$$

where the sum $j$ runs over the point charges with a charge $g_{j}$ in the selected lattice space. $\alpha_{j}, \beta_{j}$ and $\rho_{j}$ are the polar coordinates of the point charge and $x_{j}$ is $\cos \alpha_{j}$.

The structural information for calculations was taken from the results of the Rietveld profile analysis carried out with our XRD data. The effective charge for Dy ${ }^{3+}\left(g_{\mathrm{Dy}}=+1.80\right)$ was optimized in a way described earlier [23] and is close to the value found for the $Y^{3+}$ ion in $\mathrm{Y}_{2} \mathrm{O}_{3}\left(g_{\mathrm{Y}}=+1.6-1.7\right)$ [50].

The shielding factors $\sigma_{k}$ are dependent on $k$ but they have only a minor effect on the $B_{q}^{4}$ and $B_{q}^{6}$ parameter values whereas the $B_{0}^{2}$ parameter is influenced to a much greater extent. The effect of the expansion factor $\tau$ is the opposite. As a whole, the free ion radial integrals $\left\langle r^{k}\right\rangle(k=2,4,6)$ are modified by a factor of $0.8,3.5$, and 8 , respectively, when both the shielding and expansion corrections have been applied.

The point-charge calculations (table 2) yielded c.f. parameters of rank four ( $B_{0}^{4}$ and $B_{3}^{4}$ ) and $B_{0}^{6}$ close to the values obtained experimentally, whereas the $B_{0}^{2}, B_{3}^{6}$, and $B_{6}^{6}$ values were found to be too large. The calculated $B_{0}^{2}$ value is too large probably due to the purely ionic model which does not take into account covalence effects. The calculated $B_{0}^{4} / B_{3}^{4}$ ratio is closer to the ideal cubic value than the experimental ratio. The calculated sixth-rank parameter ratios are smaller whereas the experimental ones are larger than the ideal ratios (table 3). As a whole, the fourth- and sixth-rank parameters were estimated rather well taking into account the simple model.

### 3.5. Paramagnetic susceptibility of $D y O F$

The temperature dependence of the reciprocal magnetic susceptibility $\left(\chi^{-1}\right)$ of the powder sample of DyOF (figure 4) follows the Curie-Weiss law ( $\chi=14.06 /(T+9.2$ ) ) between 10 and 360 K . The calculated magnetic moment is $10.57 \mu_{\mathrm{B}}$, which agrees well with the expected value, $10.63 \mu_{\mathrm{B}}$, for the free ion ground term ${ }^{4} \mathrm{I}_{9 / 2}$ of the $\mathrm{Dy}^{3+}$ ion. Below 10 K the experimental data exhibit a deviation from the Curie-Weiss law and the reciprocal susceptibility shows a clear minimum at 3.6 K (figure 4 ). The minimum can be attributed to a three-dimensional antiferromagnetic ordering in the $\mathrm{Dy}^{3+}$ sublattice. It is not known whether structural modifications were associated with the antiferromagnetic ordering. The observed Néel temperature of 3.6 K for DyOF is considerably higher than the value reported for $\mathrm{Dy}_{2} \mathrm{O}_{3}$ [51] but identical to those obtained for $\mathrm{DyVO}_{4}$ [52] and $\mathrm{DyPO}_{4}$ [53]. In the case of SmOF , isostructural to DyOF , no magnetic ordering has been reported down to 1.5 K [54].

The analysis of the structure of DyOF can be used to explain the magnetic interactions. $\mathrm{Dy}^{3+}$ is eight coordinated to four oxygens and four fluorides (see the section on crystal structure) and thus four different $\mathrm{Dy}-\mathrm{O}(\mathrm{F})-\mathrm{Dy}$ superexchange pathways are operative. Since


Figure 4. Experimental (circles) and calculated (solid line) paramagnetic susceptibility of DyOF at temperatures between 1.7 and 370 K . The values calculated by the Curie-Weiss law $(\chi=14.06 /(T+9.2))$ are given with a dotted line.
the Dy-O distances are shorter than the Dy-F ones, it is expected that the superexchange should be more effective in the former pathway where oxygen is involved. However, due to the important deviation of the $\mathrm{Dy}-\mathrm{O}(\mathrm{F})-\mathrm{Dy}$ angles $\left(102-116^{\circ}\right)$ from the optimum superexchange angle $\left(180^{\circ}\right)$ and the shielded position of the 4 f orbitals, a value of the Néel temperature as small as 3.6 K can be anticipated.

The temperature dependence of the experimental paramagnetic susceptibility was compared with the calculated values obtained from the van Vleck formula [55]:

$$
\begin{equation*}
\chi=N_{A} \beta^{2} \sum_{i}\left(\frac{\left\langle\Phi_{i}\right| \mu\left|\Phi_{i}\right\rangle^{2}}{k T}-2 \sum_{i \neq j} \frac{\left\langle\Phi_{i}\right| \mu\left|\Phi_{j}\right\rangle\left\langle\Phi_{j}\right| \mu\left|\Phi_{i}\right\rangle}{E_{i}-E_{j}}\right) B_{i} \tag{6}
\end{equation*}
$$

where $B_{i}=\exp \left(-E_{i} / k T\right) / \sum_{i} d_{i} \exp \left(E_{i} / k T\right)$ and $\mu=\hbar(L+2 S)$.
The first-order matrix elements, $\left\langle\Phi_{i}\right| \mu\left|\Phi_{i}\right\rangle$, lead to the Curie behaviour and the secondorder ones, $\left\langle\Phi_{i}\right| \mu\left|\Phi_{j}\right\rangle\left\langle\Phi_{j}\right| \mu\left|\Phi_{i}\right\rangle$, to the temperature-independent susceptibility. $N_{A}$ and $\beta$ are the Avogadro constant and the Bohr magneton, respectively. The wave functions $\Phi_{i}$ and $\Phi_{j}$ are the eigenfunctions of the Hamiltonian corresponding to the eigenvalues $E_{i}$ and $E_{j}$ of the energy levels for the $\mathrm{Dy}^{3+}$ ion. $\mu$ is the magnetic operator, $B_{i}$ is the thermal population of the energy levels according to the Boltzmann distribution law, and $d_{i}$ is the degeneracy of the energy level.

The calculation was carried out by using the wave functions obtained from the energy level simulation. The levels up to $10000 \mathrm{~cm}^{-1}$ were accounted for in the calculation together with $J$ mixing of the levels. The calculated parallel $\left(\chi_{\|}\right)$and perpendicular $\left(\chi_{\perp}\right)$ susceptibilities show large anisotropy at low temperatures below 50 K . Good agreement between the observed and calculated susceptibility was observed above the Néel temperature (figure 4). The results indicate that the wave functions and the whole energy level simulation were correct. Moreover, the inclusion of the higher excited levels-having only negligible contribution-has not impaired the simulation of the magnetic susceptibility.

## 4. Conclusions

The optical absorption and emission spectra of the Dy ${ }^{3+}$ ion in the hexagonal DyOF and GdOF, respectively, were measured at selected temperatures down to 9 K . The analysis of the spectra yielded 153 c.f. components representing $39 S L J$ states. The analysis of the polarization and the intensity of the lines may have modified the assignments of the transitions but the former investigations could not be carried out due to the lack of singlecrystal samples. However, as a whole, the results of the c.f. analysis can be considered reliable. The energy level scheme was successfully simulated according to the $\mathrm{C}_{3 \mathrm{v}}$ site symmetry by a phenomenological model using 14 free ion and six c.f. parameters resulting in an rms deviation of $17 \mathrm{~cm}^{-1}$.

The free ion parameters resemble to those observed for $\mathrm{Dy}^{3+}$ in other $\mathrm{RE}^{3+}$ systems. The increase in the free ion parameter values from $\mathrm{Nd}^{3+}$ to $\mathrm{Sm}^{3+}$ and from $\mathrm{Sm}^{3+}$ to $\mathrm{Dy}^{3+}$ indicates increasing electrostatic repulsion and spin-orbit coupling between the 4 f electrons. The $B_{0}^{2}$ value close to zero and the c.f. parameter ratios $B_{0}^{4} / B_{3}^{4}$ and $B_{0}^{6} / B_{q}^{6}(q=3$ and 6) close to the ideal cubic values reflect the cubic pseudosymmetry of the REOF structure. Similar results were obtained for the c.f. parameters calculated by the modified electrostatic point charge model which used the XRD structure data obtained by the Rietveld refinement method.

The present results were found to be consistent with those from the previous studies concerning $\mathrm{Pr}^{3+}, \mathrm{Nd}^{3+}, \mathrm{Sm}^{3+}, \mathrm{Eu}^{3+}$, and $\mathrm{Tb}^{3+}$. In the entire REOF: $\mathrm{RE}^{3+}$ series, the phenomenological c.f. parameters decrease from $\mathrm{Pr}^{3+}$ to $\mathrm{Tb}^{3+}$. Further work is currently in progress with the heavier $\mathrm{RE}^{3+}$ ions (from $\mathrm{Ho}^{3+}$ to $\mathrm{Yb}^{3+}$ ) in the RE oxyfluoride host.

The measured magnetic susceptibility from 1.7 to 360 K follows paramagnetic behaviour close to the Néel temperature, 3.6 K , below which DyOF is ordered antiferromagnetically. The low Neel temperature can be explained by weak superexchange in the $\mathrm{Dy}^{3+}$ sublattice. By using the experimentally determined free ion and c.f. wave functions, the theoretical paramagnetic susceptibility was calculated and good agreement was found between the calculated and observed susceptibilities down to the Néel temperature.

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